END SEMESTER EXAMINATION

Business Mathematics: CMBM0079

Total Marks: 100 ***Duration: 3 hrs***

1. **Answer the following questions** [CO1] 1x10=10
   - i. Define identity matrix.
   - ii. Write down the expression for inverse of a matrix.
   - iii. If a row or a column of a determinant becomes zero then the value of the determinant will be_____
   - iv. State true or false: Multiplication of two matrices is commutative in general.
   - v. Write down the formula for amount of compound interest.
   - vi. When a function y=f(x) is said to be maximum?
   - vii. The derivative of 35x at x= 1 is _____
   - viii. When a function is said to be continuous?
   - ix. Find the integral of 1/x.
   - x. When the limit of function exists?

2. **Answer briefly *any five* of the following questions** 3x5=15
   - i. Using the properties of determinants show that $\begin{vmatrix} 1 & 1^2 & 2^2 \\ 2 & 2^2 & 4^2 \\ 3 & 3^2 & 6^2 \end{vmatrix} = 0$ [CO1]
   - ii. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$, find A^3. [CO2]
   - iii. Solve the following problem graphically
     Maximize Z=3x+9y
     Subject to the constraints
     $x + 3y \leq 60; x + y \geq 10; x - y \leq 0; x \geq 0; y \geq 0$ [CO2]
   - iv. Find the derivative of $y = x\sin(a+y)$ [CO2]
   - v. Let $f(x)$ be a function defined by $f(x) = \begin{cases} 4x-5 & , x \leq 2 \\ x-k & , x > 2 \end{cases}$. Find k if $\lim_{x \to 2} f(x)$ exists. [CO2]
   - vi. Evaluate $\lim_{x \to a} \frac{\sqrt{x+a}-\sqrt{x-a}}{x}$ [CO2]

3. Answer any *five* of the following questions [CO3] 5x7=35

i. A person deposits Rs. 10,000 in a bank which he pays an interest of 8% per annum compounded continuously. How much amount will be accumulated in his account after 5 years?

ii. A person invests money in a bank paying 6% interest compounded semi-annually. If the person expects to receive Rs. 8000 in 6 years, what is the present value of the investment?

iii. Solve the following equations by Cramer's rule $x+y+z=3$, $2x+3y+4z=9$, $x+2y-4z=-1$

iv. Find the maximum and minimum values of the following:

i. $f(x)=x^3-6x^2+9x+15$

v. If $y=\sqrt{\frac{x}{a}}+\sqrt{\frac{a}{x}}$, prove that $2xy\frac{dy}{dx}=\frac{x}{a}-\frac{a}{x}$.

vi. Evaluate $\int\frac{2x+5}{x^2+5x-7}dx$

4. Answer any *four* of the following questions [CO4] 4x10=40

i. Find the inverse of the matrix $A=\begin{pmatrix}1&2&3\\1&3&3\\1&2&4\end{pmatrix}$

ii. Using properties of determinant prove that $\begin{vmatrix}1&a&a^3\\1&b&b^3\\1&c&c^3\end{vmatrix}=(a-b)(b-c)(c-a)(a+b+c)$

iii. Using properties of the determinant show that $\begin{vmatrix}x&y&z\\x^2&y^2&z^2\\x^3&y^3&z^3\end{vmatrix}=xyz(x-y)(y-z)(z-x)$.

iv. A machine costing Rs. 50000 depreciates at a constant rate 8%. What is the depreciation charge for the 8th year? If the estimated useful life of the machine is 10 years, determine the scrap value.

v. Evaluate the following:

a) $\lim_{x\to5}\frac{2x^2+9x-5}{x+5}$

b) $\lim_{x\to\frac{1}{4}} \frac{4x-1}{2\sqrt{x}-1}$

vi. (a) Suppose $f(x)=\begin{cases} a+bx & ,x<1 \\ 4 & ,x=1 \\ b-ax & ,x>1 \end{cases}$ and if $\lim_{x\to 1} f(x)=f(1)$, what are the possible values of *a* and *b*?

(b) If for $f(x)=\lambda x^2+\mu x+12$, $f'(4)=15$ and $f'(2)=11$, find λ and μ.

vii. Evaluate the following integrals:

a) $\int\left(x^2+\frac{1}{x^2}\right)^3 dx$

b) $\int x^2 e^x dx$